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BAYESIAN INFERENCE OF GROUTED ANCHORS FOR RELIABILITY ANALYSES

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Bayesian analysis is a powerful tool to quantify uncertainty. Based on a prior estimate of a parameter distribution, and a likelihood function which ties together the simulated model response with the data, one estimates the posterior distribution. Therefore, this approach enables the calculation of the data distribution itself given prior knowledge and limited data. In geotechnical engineering, this asset is of special interest, since data in most applications is scarce.

There is limited knowledge about the bearing behavior of grouted anchors at design stage, which entails that in practice many anchor tests are conducted to ensure and verify that an anchor can provide its anticipated resistance. The design of an anchored structure is usually not altered unless several anchors fail the testing, thus apart from verification the testing data yields limited additional value.

This research introduces and compares different statistical models which allow failure probability estimation of individual grouted anchors and anchor groups. An analytical anchor-head displacement model is utilized to assess the anchor test data with a multivariate normal distribution as a likelihood function with weakly informative prior distributions. The soil parameters are treated as random variables. The statistical models are purely data-driven, meaning that no prior knowledge of the soil parameters is utilized, apart from upper limits. The individual statistical model allows the analysis of a single anchor for a specific load step by directly calculating posterior distributions for the uncertain soil parameters. Anchor groups are assessed using a hierarchical model formulation, where the distribution of uncertain parameters and the corresponding hyperparameters are treated as unknowns. These analyses allow inferences about probability of failure and expected resistance distributions.

The statistical models deliver failure probabilities, that are in line with the data distribution itself, and thereby provide a method for the reliability assessment of grouted anchors. A major influential factor in the reliability assessment is the modelling choice of the likelihood function’s covariance. Furthermore, the quality of measurements taken is governing the quality of the inferred anchor’s resistance estimation.

Keywords: Bayesian; anchor; reliability; hierarchical; probabilistic.

1. Introduction

Grouted anchors are commonly used in geotechnical engineering structures like deep excavations and quay walls. Their bearing behavior is highly non-linear and uncertain to such a degree, that according to European guidelines an unprecedented testing effort needs to be excerpted for the verification of their design bearing capacity. Nevertheless, the obtained data is used seldomly for other purposes and the design is usually only altered when many anchors fail. Additionally, different sources of uncertainty, like spatial variability, afflict the anchor data and complicate the test evaluation. In recent years, probabilistic methods became more popular in geotechnical engineering to cope with uncertainty and aid in the decision-making process. Currently, there is no method that enables a more precise design of grouted anchors than the semi-empirical methods employed since the 1970s by Ostermayer (1975) and Littlejohn (1970), nor is there a framework that deals with anchor test result variability. The aim of this research is to provide a data-driven Bayesian method that enables the probability of failure estimation and the pull-out resistance estimation based on suitability and investigation test data.

In literature, there are different examples for applications of Bayesian analyses in geotechnical engineering. The bearing capacity of piles and pile groups can be updated using Bayes (Huang et al., 2016; Lacasse et al., 1989, Nietiedt et al., 2019) as for the factor of safety of slopes (Conteras & Brown, 2019, Jiang et al., 2018, 2022). Depending on the structure of the data, hierarchical modelling techniques can be employed as illustrated by Bozorgzadeh & Bathurst (2020) for mechanically stabilized earth walls, for the estimation of geotechnical parameters (Mavritsakis et al.; 2022) or the updating of random field properties for slope analyses (Geyer et al., 2021). The foundations of reliability analysis with reference to geotechnical engineering can be related in Baecher & Christian (2003), while Gelman et al. (2013) conveys the basic concepts regarding Bayesian analysis.

2. Methods

Bayesian analysis revolves around the approximation of Bayes Theorem. It describes the relationship on how data is utilized in the combination with prior knowledge to determine the posterior distribution of a random variable ¸ . Bayesian analysis is an effective modeling technique in combination with scarce data. The aim is to use the data to make an improved estimate of the distribution of a random variable by combining it with prior knowledge. The resulting posterior distribution approximates the true data distribution for a lot of data (Gelman et. al., 2013). Depending on the problem statement, a Bayesian analysis can be computationally expensive since closed form analytical solutions are generally only available from conjugate formulations. Thus, sampling algorithms are commonly employed to approximate the posterior distribution. In the scope of this study the results obtained by MT-DREAM$_{zs}$ algorithm (Laloy & Vrugt, 2012) are presented.

Bayes Theorem for the posterior distribution P_D is defined as,

$$P_D(\theta|D) = \frac{L(\theta|D) \cdot P_0(\theta)}{\int L(\theta|D) \cdot P_0(\theta)} \tag{1}$$

Here, ¸ are the random variables, *D* is the data, P_0 denotes the prior distribution, *L* is the likelihood function. The following list summarizes the modelling assumption regarding these quantities:

- ¸ , random variable vector, here model parameters of an auxiliary rheological anchor displacement model
- *D*, anchor test data, measured displacement over time of anchor suitability and investigation tests, their testing procedure is regulated in ISO-22477-5
- P_0, prior distribution, weakly informative priors as uniform distributions with hard upper bounds for both models
- *L*, likelihood function, multivariate normal distribution with mean value (*D*) and covariance (£)

One anchor test comprises the measured displacements over time for several increasing load steps. The Bayesian analysis focuses on anchors and anchor groups, which are loaded under similar mean shear stress along the grouted body. To tie the measured anchor head displacements of a load step to a posterior distribution in a statistical model, an auxiliary analytical model is necessary. In literature, one of the few analytical models capable of depicting anchor head displacements over time, was identified to be the model of Montero-Cubillo (2020). The anchor head displacement is partitioned into contributing displacement components, the grout body displacement, the tendon elongation and the displacement of the retained structure against the soil due to the loading. The model works with the soil shear modulus *G* and Poisson's ratio ½as input variables regarding soil strength. For the calculation of displacement over time, these soil parameters are transformed into time dependent functions *G(t)* and *½(t)* based on the rheology of the Burger model, introducing the Maxwell shear modulus, Maxwell viscosity, Kelvin-Voigt shear modulus, Kelvin-Voigt viscosity. Within the Bayesian framework, these rheological soil parameters are treated as unknowns and modelled as random variables. For more information on the analytical model, the reader is referred to the source itself.

To perform Bayesian inference, the analytical model *M* must be incorporated into the statistical, Bayesian model. As already mentioned, the analytical model *M* functions as a bridge to obtain posterior distributions corresponding to the measured quantity *D*. The model itself is interchangeable, numerical models can be more precise, but their direct employment is not feasible as part of the likelihood function due to the long computation times, without considering surrogate- or meta-modeling. The measurements and the model response are compared in the likelihood function *L*, a multivariate normal distribution.

$$L(\theta|D) = \frac{e^{-\frac{1}{2}\cdot(M(\theta)-D)^T\cdot\Sigma^{-1}\cdot(M(\theta)-D)}}{\sqrt{(2\pi)^k|\Sigma|}} \tag{2}$$

The data *D* is the mean value of the function, £ the covariance matrix, and *k* the dimension. Depending on the statistical model, individual or hierarchical, the dimensions of the likelihood function input changes. For the individual model, where a single anchor is considered, the data is anchor head displacement over time, a vector. For the hierarchical model, the data are a group of anchor head displacement measurements over time, a matrix.

In the individual model, the random variables ¸ are the rheological soil parameters, terms to model the likelihood's covariance as a diagonal, and two optional terms, the creep value k_s and the R^2, for better results regarding the posterior-predictive creep value distribution. In the hierarchical model, the key assumption is that the uncertain rheological soil parameters of an anchor group in the same soil conditions can be described by a single distribution and can therefore be sampled from this distribution accordingly. Therefore, the uncertain parameters are not modeled directly but indirectly in the form of the hyperparameters of said distribution: mean value and standard deviation. The presumed parameter distributions for rheological soil parameters and covariance are lognormal, the standard deviations are modeled as fractions of the mean value to avoid sampling standard deviations that are larger than the mean value. The marginal posterior distributions of the random variables ¸ are the direct result of the Bayesian analysis. Since the interpretation of marginal distributions is difficult regarding the bearing behavior of the anchor. More tangible is the resulting posterior-predictive (pp) distribution, which is the same quantity as the data *D*, displacements over time. Using the pp-distribution, the pp creep value distribution is calculated, all for each loadstep separately. The creep value is the half-logarithmic gradient of the displacement curve between two time steps and its magnitude is the criteria for the geotechnical failure assessment of an anchor. An anchor is considered failed once the creep value is larger than or equal to 2.0 mm. The probability of failure is estimated using the creep value distribution samples according to Betz et. al. (2022) and depicted as the 95% credible interval. Since the creep value evolution for increased load is supposed to follow an exponential trend, an exponential function is fitted to the creep value distribution samples of each load step. Based on the exponential curves and their intersection with the failure limit at 2.0 mm creep, the predictive pull-out capacity distribution is estimated.

3. Results

The individual model enables an estimation of the anchor's probability of failure, considering the total predictive uncertainty, consisting of model and observation uncertainty. Fig. 1a shows the pp-distribution of a representative anchor of case study 1 (9 anchors in total), with its corresponding pp creep value distribution samples (Fig. 1b). The hierarchical model exhibits similar results, but the pp-distribution and the creep value distribution are valid for all anchors of the investigated group (Fig. 1c + Fig. 1d). The pp-distribution of the individual model compared to the hierarchical model exhibits larger variability. Additional to observation and model uncertainty, the hierarchical model inherently captures a spatial uncertainty component consisting mainly of installation effects and spatial variability as well. In a hierarchical model, based on the pp-distribution, the measurements' probability of occurrence can be estimated to determine if particular measurements are outliers.

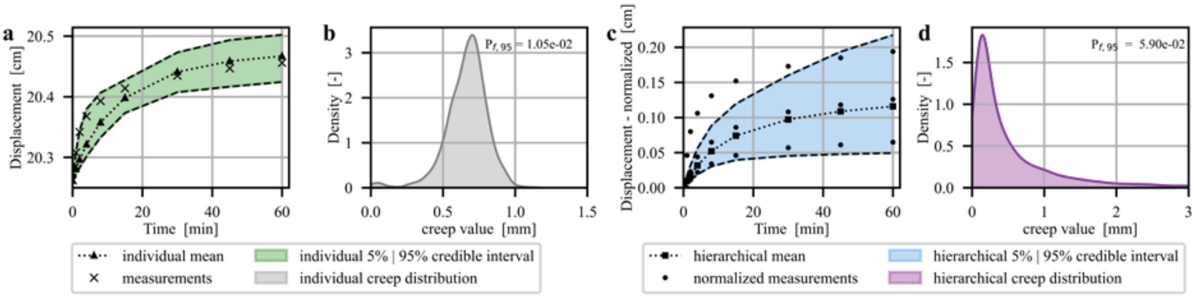

Fig. 1. Posterior-Predictive (**a**+**c**) and creep value distribution (**b**+**d**) for individual and hierarchical model.

Figure 2 shows results of case study 2 (3 anchors in total), measured creep measurements for increasing load of suitability tests, which have been extended to investigation tests at a later stage in the project. Fig. 2a and Fig 2b show the creep extrapolation results and the corresponding pull-out capacity distribution respectively. Fig 2c shows the creep extrapolation for the hierarchical model investigation and suitability tests with mean, 5% and 95 % credible intervals. Extrapolations yield a high degree of uncertainty especially since the creep value itself is ambiguous. Naturally, such an extrapolation works best if the measurements exhibit the presumed exponential trend, however real data does not always show this trend.

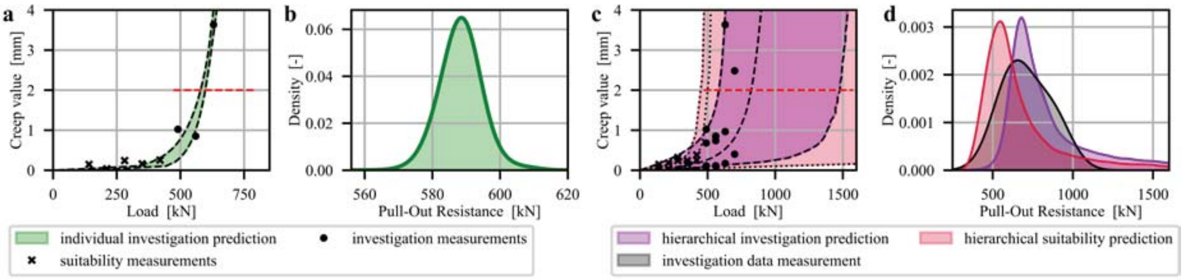

Fig. 2. Creep value extrapolation (**a**+**c**) and predictive resistance distribution (**b**+**d**) for individual and hierarchical model.

In Fig. 2d, the measured pull-out resistances are illustrated as a distribution, which is compared to the prediction conducted after the suitability tests. The prediction is in line with the measurements, having a larger variance than the measured data distribution, but with similar mean values and weak tails. The purple curve is the updated distribution of the resistance after conducting the investigation tests. The updated distribution provides more certainty about the mean value and the tails of the distribution, enabling a more efficient design than just relying on the scarce data itself.

4. Conclusions

The inference for individual and hierarchical models are different because of the underlying uncertainty and its sources considering the prediction. The individual model aims to depict observation and model uncertainty of anchor head displacements above other minor sources of uncertainty. The hierarchical model additionally incorporates a spatial uncertainty component including spatial variability and installation effects. Therefore, the spread in the prediction is larger for the hierarchical model. The presented procedure shows, that the probability of failure of anchors and anchor groups can be assessed without an additional testing program, based solely on suitability or investigation test results. The resulting probabilities of failure can be comparatively large, for anchor groups where several anchors failed the load test or individual anchors whose measured creep value is close to the failure limit. This probability of failure needs to be understood as “unsatisfactory performance” rather than “catastrophic failure” (Duncan, 2000). Using these inferences, the creep value evolution for increasing load can be estimated, enabling the extrapolation to approximate a predictive pull-out resistance distribution. The initial investigations show that the predictive resistance distribution is in line with measured resistances, validating the framework. Generally, this Bayesian analysis enables inferences about the anchors bearing behavior and reduces uncertainty. Currently, larger databases and different statistical modeling approaches, especially regarding the covariance, are under investigation to isolate different sources of uncertainty, and provide further validation and verification of the proposed data-driven approach. The presented framework is currently being validated with different data sets and is going to be tested for reliability analysis of different kinds of retaining structures.

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